Gravitational Decoherence for Timed Dicke State

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Introduction

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This is the reason for popularity of that topic now.

Timed Dicke State

N stationary identical two-level atoms interact with the electromagnetic field.

The Hamiltonian:

$$\hat{V}(t) = \sum_{k,j} \hbar \left(v_k^* \hat{\sigma}_j^\dagger \hat{q}_k e^{-i(\nu-\omega_k)t+i\mathbf{k}\mathbf{r}_j} + h.c. \right). \tag{1}$$

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So,

$$|\Psi(0)\rangle = |g, 1_{k_0}\rangle \implies \qquad |\Psi\rangle_{dicke} \approx \frac{1}{\sqrt{N}} \sum_j e^{i\mathbf{k}_0\mathbf{r}_j} |e_j, 0\rangle^1.$$

1 M. O. Scully, E. S. Fry, C. H. Raymond Ooi, and K. Wodkiewicz, *Directed Spontaneous Emission from an Extended Ensemble of N Atoms: Timing Is Everything*, Phys. Rev. Lett. **96**, 010501 (2006).

Timed Dicke State

After spontaneously decay:

$$|\Psi(\infty)\rangle = \frac{1}{\sqrt{N}} \sum_{j,k} \frac{v_k e^{i(\mathbf{k}_0 - \mathbf{k})\mathbf{r}_j}}{(\omega_k - \nu) + i\frac{\Gamma}{2}} |g, 1_k\rangle. \tag{2}$$

Changing the summation over all atoms to the integration of the volume gives the Dirac delta-function:

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Photon is emitted into initial direction!

Considering of gravitation

We will take into account the gravitation field by using of the weak field approximation of the Schwarzschild metric:

$$ds^2 = (1 + a(z - z_0))\, c^2 dt^2 - \left(dx^2 + dy^2 + (1 - a(z - z_0))\, dz^2\right),$$

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So, what will happen with electromagnetic field in that case?

Electromagnetic wave in the presence of gravitation

The solution of Maxwell equations in flat space is represented by

$$E_i = \sum_{\mathbf{k}} \sum_{s=1}^{2} \alpha_{\mathbf{k}} \left[\hat{q}^*_{\mathbf{k}s} f_i(\mathbf{k}, s) e^{i\Theta} + h.c. \right],$$

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It is found that for each $\mathbf{k}^{(0)} = \{k_x, k_y, k_z\}$:

$$\alpha_{\mathbf{k}} \approx \left(\frac{\hbar \omega_{\mathbf{k}}}{2\varepsilon_0 V_0} \right)^{1/2} \left(1 + a(z - z_0) \frac{k_x^2 + k_y^2}{4k_z^2} \right),$$

$$\Theta \approx c\sqrt{k_x^2 + k_y^2 + k_z^2}\, t - k_x x - k_y y - k_z z + a \frac{k_x^2 + k_y^2 + 2k_z^2}{4k_z} (z - z_0$$

$$\tilde{k}_\mu \approx \left\{ c\sqrt{k_x^2 + k_y^2 + k_z^2}, -k_x, -k_y, -k_z + a \frac{k_x^2 + k_y^2 + 2k_z^2}{2k_z} (z - z_0) \right\}$$

Electromagnetic wave in the presence of gravitation

and for 3-vector of polarization one has

$$
\begin{aligned}
f_1(\mathbf{k},s) &\approx f_1^{(0)}(\mathbf{k},s) + a\frac{z-z_0}{2}\frac{k_x}{k_z}f_3^{(0)}(\mathbf{k},s),\\
f_2(\mathbf{k},s) &\approx f_2^{(0)}(\mathbf{k},s) + a\frac{z-z_0}{2}\frac{k_y}{k_z}f_3^{(0)}(\mathbf{k},s),\\
f_3(\mathbf{k},s) &\approx f_3^{(0)}(\mathbf{k},s),
\end{aligned}
$$

where $f_i^{(0)}(\mathbf{k},s)$ is the 3-vector of polarization for the flat space.

Electromagnetic wave in the presence of gravitation

The Hamiltonian of free electromagnetic wave is

$$\hat{H} = \hbar \sum_{\mathbf{k}} \omega_{\mathbf{k}}(Z) q_{\mathbf{k}}^{\dagger} q_{\mathbf{k}}.$$

where $\omega_{\mathbf{k}}(Z) = \omega_{\mathbf{k}}(1 + a(Z - z_0)/2)$.

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Is it trivial?

Atom-field interaction in the presence of gravitation

The Hamiltonian of just one two-level atom system in the interaction picture is

$$\hat{V}(t) = \sum_k \hbar \left(v_k^*(r_{at}) \hat{\sigma}^\dagger \hat{q}_k e^{i(\nu - \omega_k(z))t} + h.c. \right),$$

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A standard equation for the for the excited state amplitude:

$$\dot{c}_{a,0}(t) \approx -\frac{\Gamma(z)}{2} c_{a,0}(t),$$

where the position-dependent decay rate is

$$\Gamma(z) = \Gamma_0 \left(1 - \frac{a(z - z_0)}{2} \right).$$

Timed Dicke state in curved time-space

Timed Dicke state in curved time-space:

$$|\Psi\rangle_{dicke}^{curv} \approx \frac{1}{\sqrt{N}} \sum_j e^{i\mathbf{k}_0\ \mathbf{r}_j} \left(1 + aF_{\mathbf{k}_0}(z_j)\right) |e_j, 0\rangle, \tag{3}$$

where $\mathbf{k}_0$ is the wave-vector of incident photon. The function $F_{\mathbf{k}_0}(z_j)$ is

$$F_{\mathbf{k}_0}(z_j) = (z_j - z_0)\frac{k_x^2 + k_y^2}{4k_z^2} - i(z_j - z_0)^2 \frac{k^2 + k_z^2}{4k_z}, \tag{4}$$

The dephasing

After emitting:

$$|\Psi(\infty)\rangle^{curv} \approx \frac{1}{\sqrt{N}} \sum_{j,k} v_{k_0} \times \left(\frac{e^{i(\mathbf{k}_0 - \mathbf{k})\mathbf{r}_j}}{(\omega_k(z_j) - \nu) + i\frac{\Gamma(z_j)}{2}} \right) |g, 1_k\rangle +$$

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$$+ a \frac{1}{\sqrt{N}} \sum_{j,k} v_{k_0} \frac{e^{i(\mathbf{k}_0 - \mathbf{k})\mathbf{r}_j}}{(\omega_k - \nu) + i\frac{\Gamma_0}{2}} F_{\mathbf{k}_0}(z_j)|g, 1_k\rangle \approx$$

$$\approx \frac{\sqrt{N}}{V_0}(2\pi)^2 \sum_k \delta(k_{x0} - k_x)\delta(k_{y0} - k_y) \times$$

$$\times \int dz \frac{v_k e^{i(k_{z0} - k_z)z}}{(\omega_k - \nu - i\frac{\Gamma_0}{2}) + \frac{a}{2}(z_0 - z)(\omega_k - i\frac{\Gamma_0}{2})} |g, 1_k\rangle,$$

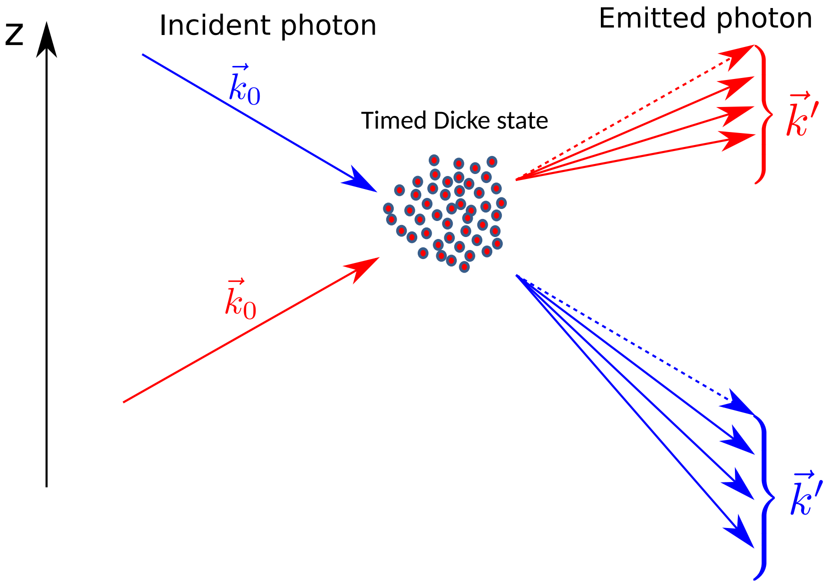
The dephasing

$$|\Psi(\infty)\rangle^{curv} = \frac{\sqrt{N}}{V_0}(2\pi)^3 \sum_k \delta(k_{x0} - k_x)\delta(k_{y0} - k_y) v_k \times$$
$$(G_k(a) + \delta(k_{z0} - k_z)O(a))|g, 1_k\rangle.$$

where for $\Gamma_0 \ll \nu$ we have

$$G_k(a) \approx \frac{-i}{a\nu} e^{-(k_{z0}-k_z)\frac{\Gamma_0}{a\nu}}. \quad (5)$$

The dephasing



Thank you for attention!