12.4

It is desired to make an $\mathbf{E} \times \mathbf{B}$ velocity selector with uniform, static, crossed, electric and magnetic fields over a length L . If the entrance and exit slit widths are Δx, discuss the interval Δu of velocities, around the mean value $u = \dfrac{cE}{B}$, that is transmitted by the device as a function of the mass, the momentum or energy of the incident particles, the field strengths, the length of the selector, and any other relevant variables. Neglect fringing effects at the ends. Base your discussion on the practical facts that $L \sim$ few meters, $E_{max} \sim 3 \times 10^6$ V/m, $\Delta x \sim 10^{-3} - 10^{-4}$ m, $u \sim 0.5 - 0.995c$. (It is instructive to consider the equation of motion in a frame moving at the mean speed u along the beam direction, as well as in the laboratory.) References: C. A. Coombes et al., *Phys. Rev.* **112**,1303 (1958); P. Eberhard, M. L. Good, and H. K. Ticho, *Rev. Sci. Instrum.* **31**, 1054 (1960).

Solution

Solution, Problem 12.4

In CGS units, the force experienced by the particle is:

$$\overrightarrow{F} = q(\overrightarrow{E} + \frac{\overrightarrow{v}}{c} \times \overrightarrow{B})$$

Let's consider:

$$\overrightarrow{v} = v\widehat{z}, \quad \overrightarrow{E} = E\widehat{x}$$

and

$$\overrightarrow{B} = B\widehat{y}$$

then

$$\overrightarrow{F} = q(\overrightarrow{E} + \frac{\overrightarrow{v}}{c} \times \overrightarrow{B}) = qE(1 - \frac{v}{c}\frac{B}{E})\widehat{x}$$

Following suggestion let $u \equiv \frac{cE}{B}$, then

$$\overrightarrow{F} = qE(1 - \frac{v}{u})\widehat{x}$$

so if

$$v = u$$

the particle is not at all deflected; contributions from $\overrightarrow{E}$ and $\overrightarrow{B}$ in the Lorentz force compensate each other. If $v = u + \Delta u$ then $\overrightarrow{F} = qE(1 - \frac{u+\Delta u}{u})\widehat{x} = -qE\frac{\Delta u}{u}\widehat{x} = \mathtt{F}\widehat{x}$. The equation of motion is:

$$\gamma m\ddot{x} = \mathtt{F} \Rightarrow \quad \ddot{x} = -\frac{qE}{\gamma mu}\Delta u\widehat{x}$$

or upon integration

$$x = -\frac{qE}{\gamma mu}\Delta u\frac{t^2}{2} + x_0$$

$t = \frac{L}{u}$ and let $\Delta x = x - x_0$ then the particle displacement is:

$$|\Delta x| = \frac{qEL^2}{2\gamma mu^3}\Delta u$$

so $\Delta u = |\Delta x|\frac{2\gamma mu^3}{qEL^2}$ for the range of parameters suggested, I got $\Delta u \approx [10^{-6} - 10^{-5}]$.