Crystalline beams

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Introduction

Definition

Coulomb crystal is a "solid" phase of charged particles (electrons, protons, ions) which are cooled to form a stable lattice.

Example: Ions in ion traps

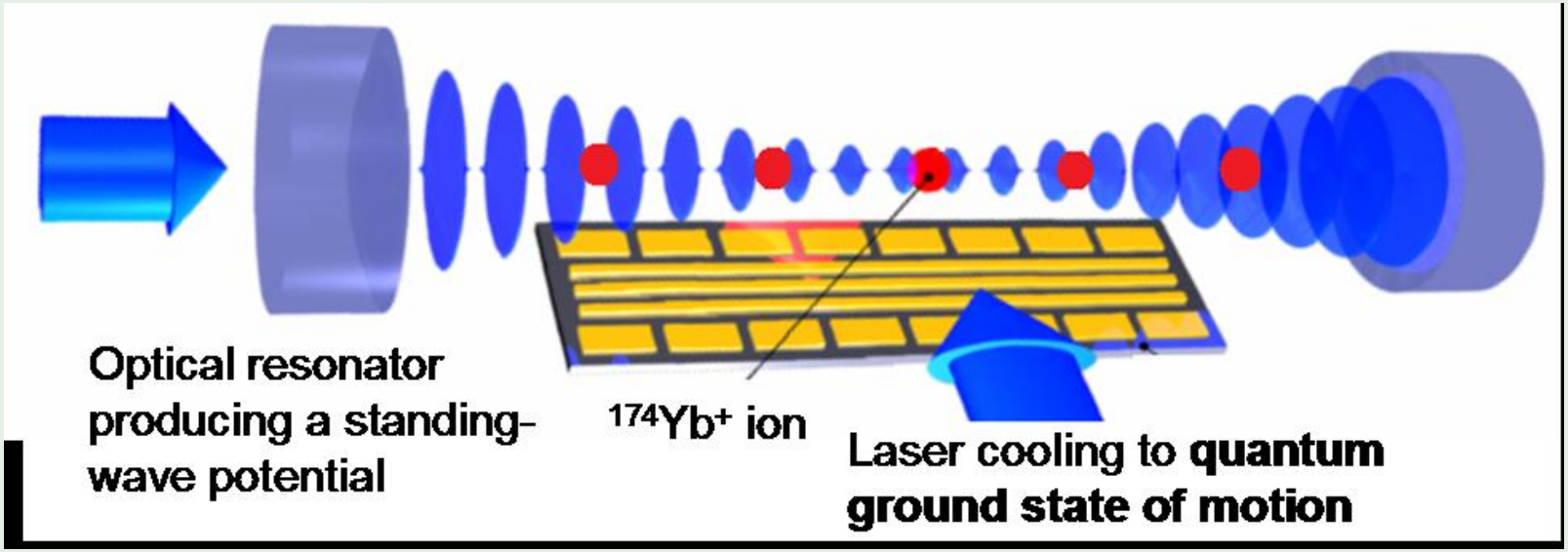


Image courtesy of A. Bylinskii

History

Discovery

- Wigner, E. *On the Interaction of Electrons in Metals.* Phys. Rev. 46, 1002 (1934).

Theoretical studies

- Schiffer, J.P. PRL 57 (1986) 1133.
- Wei, J. PRL 73 (1994) 3089.
- Hasse, R.W. PRL (1999) 3430.

Observations

- Parkhomchuk, V. at NAP-M (Russia, 1979)
- Steck, M. at ESR (Germany, 1996)
- Schatz, T. at PALLAS (Germany, 2001)

Beam requirements

Temperature

$$\Gamma = \frac{U}{T} = \frac{1}{4\pi\epsilon_0}\frac{Z^2e^2}{aT} > 1$$

Strong focusing

The storage-ring must be strong focusing and the beam energy must be below the transition energy

Absence of resonance

There must be no linear resonance between crystal phonon modes and the machine lattice periodicity

Coulomb crystals

Crystalline states

- Ordered beam - string of charges
- Zigzag beam - 2D states
- Helical beam, etc. - 3D states

1D infinite crystal

$$M\ddot{z}_n + \sum_{k=1}^{\infty} \frac{Q^2}{(z_n - z_{n+k})^2} - \sum_{k=1}^{\infty} \frac{Q^2}{(z_n - z_{n-k})^2} = 0$$

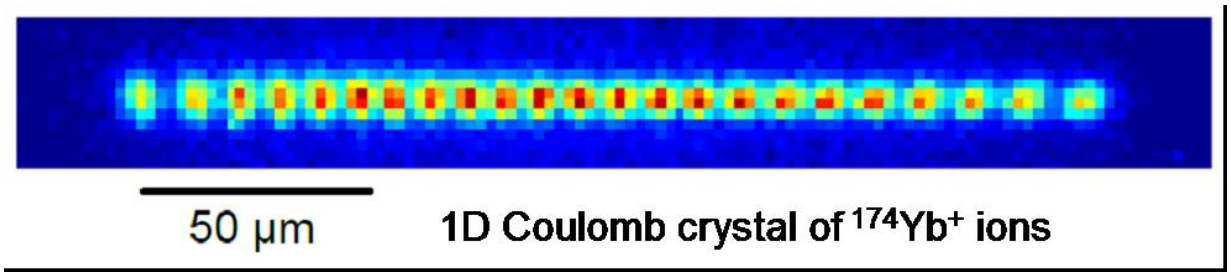


Image courtesy of A. Bylinskii

Eigenvalue problem

The equation above can be linearized as

The longitudinal location of the $n-th$ particle is designated as $z_n = na + \Delta_n$, where Δ_n is a small displacement from its lattice site.

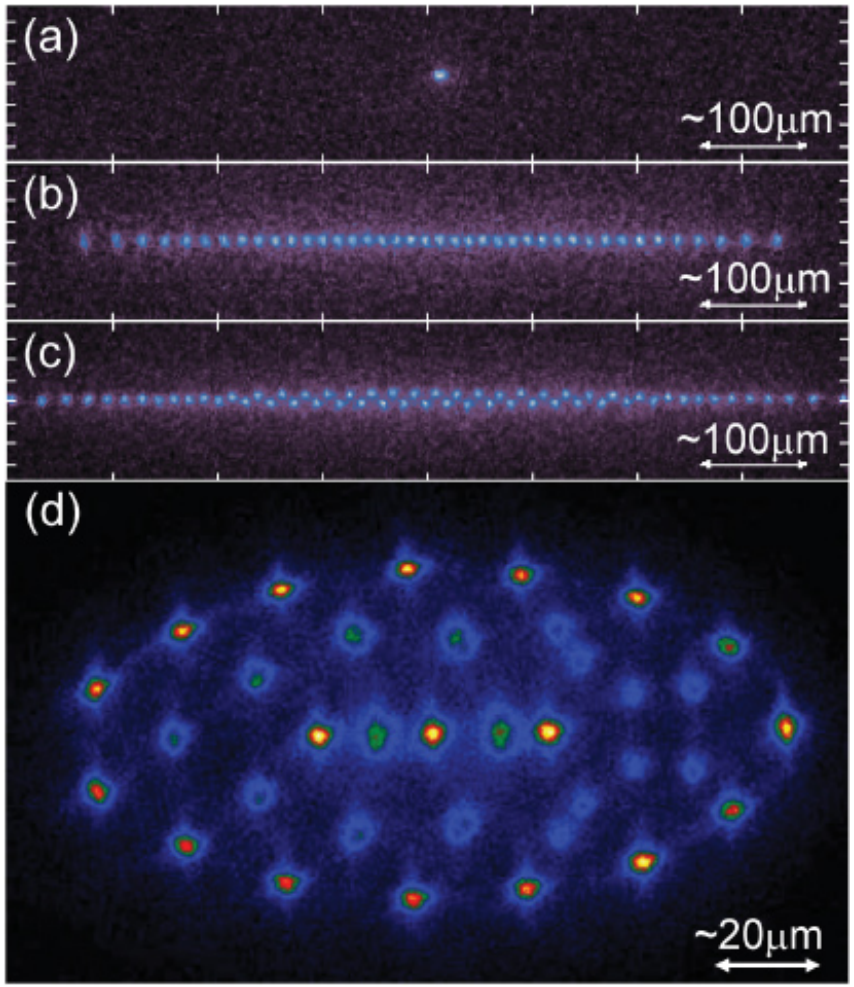
$$M\ddot{\Delta}_n + \sum_{k=1}^{\infty} \frac{2Q^2}{k^3 a^3}(2\Delta_n - \Delta_{n+k} - \Delta_{n-k}) = 0$$

Eigenmodes and eigenvectors

$$\lambda = 4\sum_{k=1}^{\infty} \frac{1}{k^3} sin(\frac{k\theta}{2})$$

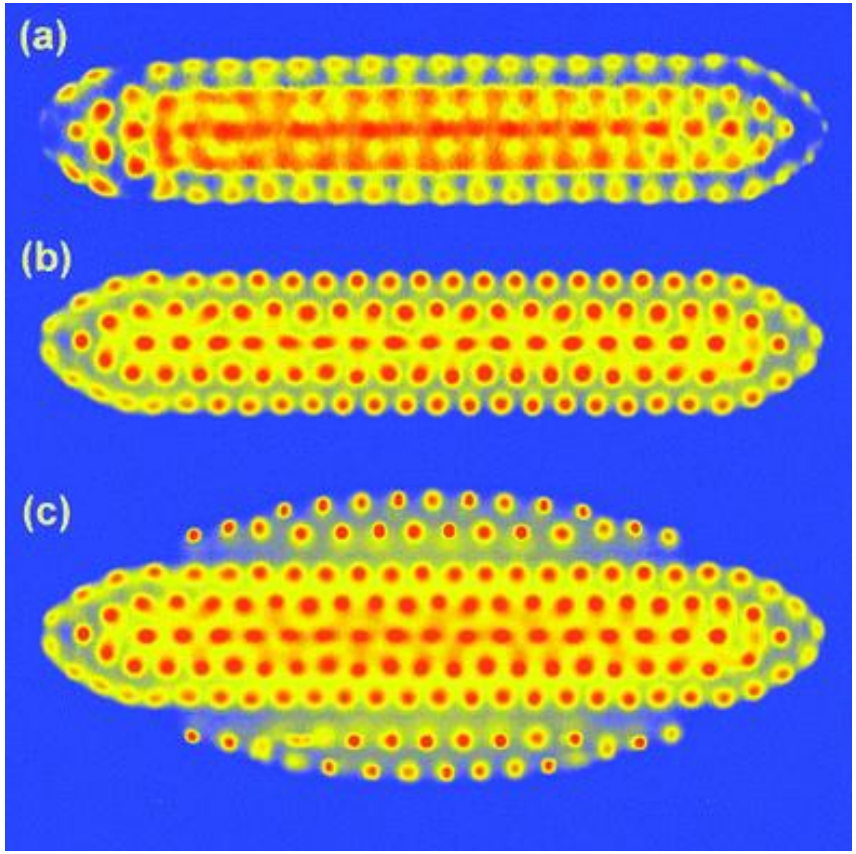
$$\Delta_n = cos(n\theta + \phi)e^{-i\omega t}$$

Higher dimensions (Mg^+)



Ch. Schneider et. al. 2012 Rep. Prog. Phys. 75 024401

False-colour fluorescence images of a $Ca+/CaF+$ bi-component Coulomb crystal



Phys. Chem. Chem. Phys., 2008,10, 7200-7210

Applications

Colliders

Tremendous increase in luminosity

Accelerators

Non-dispersive lattice elements, test of fields homogeneity.

Quantum solid state physics

Great venue for experimental tests of various models

Quantum computing

Exploit new features of low-dimensional quantum systems